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A Diagramatic Analysis of Duality in Supersymmetric Gauge Theories

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Abstract

We introduce a diagramatic notation for supersymmetric gauge theories. The notation is a tool for exploring duality and helps to present the field content of more complicated models in a simple visual way. We introduce the notation with a few examples from the literature. The power of the formalism allows us to study new models with gauge group $(SU(N))^k$ and their duals. Amongst these are models which, contrary to a naive analysis, possess no conformal phase.

1 Introduction

Following the initial work of Seiberg on supersymmetric QCD (SQCD) [1] many examples of duality in supersymmetric $N = 1$ field theories have been discovered. More recently, various groups have studied more complicated models with several gauge and global symmetries and large numbers of fields [2, 3]. The commonly used notation showing tables of fields and representations often proves to be very cumbersome. We propose a diagramatical notation that shows how the fields of the theory under consideration transforms under gauge and global symmetries.

The notation is based on a notation introduced by Georgi [4] for the study of non-supersymmetric strongly coupled gauge theories (also used in the models of [5]). In supersymmetric theories the exact results on the infrared spectrum of strongly coupled theories are particularly easily displayed using our "Duality Diagrams". Seiberg's dualities have a particularly transparent realization in this notation. A simple set of "Duality Rules" which would be amenable to implementation on a computer enable one to derive dualities of more complicated product group theories. The proposed notation can be used for SU, SP, and SO theories with matter in the fundamental and tensor representations. We will concentrate mainly on SU theories with fundamental matter in this paper. In section 2, we introduce our notation using SQCD as an illustrative example. The next simplest model involves two SU gauge groups and has been previously studied in [3]. We reanalyze this model in section 2.2 as an example of the diagramatic notation.

In section 4, we use our notation to study new models with matter transforming under an $(SU(N))^k$ gauge group. When all possible Yukawa interactions consistent with the gauge symmetry are included the model has an $SU(F)^k$ global symmetry. We find a dual description in terms of an $SU((F-N))^k$ gauge theory and a similar superpotential. For $F \geq 2N$ the original theory is infrared free, whereas for $F \leq 2N$ the dual is free in the infrared. The theory therefore has no conformal phase. At the special point $F = 2N$ the two descriptions coincide as required for consistency. The dual theory allows one to completely understand the infrared dynamics of the very complicated strongly coupled electric theory for $N + 1 < F < 2N$. We also discuss the model without Yukawa couplings which does have a conformal phase.

Finally, we present Duality Diagrams for theories with tensors and with SO and SP gauge groups.

2 SQCD with Duality Diagrams

We introduce our notation using the "classic" example of $N = 1$ SQCD and its dual discovered by Seiberg [1]. In SQCD, the electric theory, is an $SU(N)$ gauge theory with F flavors of "quarks" in the fundamental and antifundamental representations of the gauge group. Thus the global symmetries of the theory include two factors of

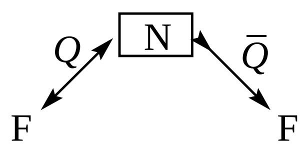
$SU(F)$ acting on the quarks and antiquarks. The tree level superpotential is taken to vanish. Frequently, the field content and symmetries of supersymmetric theories are displayed using tables of Young tableaux. For example, the table for SQCD is

field	$SU(N)$	$SU(F)$	$SU(F)$
Q	$\square$	$\square$	$\mathbf{1}$
$\bar{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$

(2.1)

In the following we will frequently use such tables to explain our notation and demonstrate it's convenience. In our diagramatical notation we denote gauge groups by boxes with the size of the gauge group stated inside the box $\boxed{\mathbf{N}}$. Non-Abelian global "flavor" symmetries are represented by the number of flavors $\mathbf{F}$. Fields are represented by lines connecting the gauge and global symmetries. For example, the quarks of SQCD are denoted by a line connecting the box for SU(N) with the F of the flavor group. Arrows on the ends of each line distinguish representations and their complex conjugates, fundamentals have incoming arrows whereas anti-fundamentals are denoted with outgoing arrows.

The diagram for SQCD is then



(2.2)

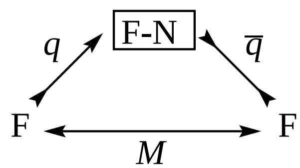
where the quark and antiquark fields are labeled Q and $\overline{Q}$, respectively. From the direction of the arrows one sees that Q transforms as a fundamental under both the $SU(N)$ gauge group and the $SU(F)$ global symmetry group. The $\overline{Q}$ is an antifundamental under $SU(N)$ and a fundamental under $SU(F)$.

According to Seiberg's conjecture, when $F \geq N+2$, this theory is dual to an $SU(F-N)$ gauge theory with F flavors of dual quarks q and $\overline{q}$ and a gauge singlet field M which corresponds to the bound state $Q\overline{Q}$ of the original theory. The "meson" M is coupled via a tree level superpotential $W = qM\overline{q}$. In terms of Young tableaux, this is

field	$SU(F-N)$	$SU(F)$	$SU(F)$
q	$\square$	$\overline{\square}$	$\mathbf{1}$
$\bar{q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$
M	$\mathbf{1}$	$\square$	$\square$

(2.3)

The diagram corresponding to this table is easily constructed. There is a box for the gauge symmetry and two $\mathbf{F}$'s for the flavor symmetries. Then there are two lines connecting the gauge and flavor symmetries corresponding to q and $\overline{q}$. Finally, there is the meson line, M, connecting the two flavor groups. The diagram is



(2.4)

Note the orientation of the arrows: the arrows corresponding to the global groups' representations on the two quark and antiquark lines are now reversed, the arrows for the gauge symmetry remain the same. The arrows for the representation of the meson field under the global groups are pointing in the direction in which those arrows for the fields Q and $\overline{Q}$ were pointing in the original model.

We can now identify a set of "Duality Rules" which enable us to write down the dual diagram starting from the diagram for the electric theory. First, the global symmetries of the model are left unchanged whilst the gauge symmetry being "dualized" is replaced with an as yet unknown dual gauge group. The lines corresponding to the dual quarks q and $\overline{q}$ are obtained by copying the lines corresponding to the original quarks Q and $\overline{Q}$ but reversing the arrows for the global groups. There is a meson field M in the dual for every pair of in- and out-going arrows of the gauge group. Here there is one incoming arrow from Q and one outgoing from $\overline{Q}$. The meson line directly connects the two groups on which the Q and $\overline{Q}$ lines ended. The orientation of the arrows on M are identical to the orientation of the arrows on the quark lines in the orginal model. This rule is easy to understand by remembering that M is the operator map of the composite gauge invariant $Q\overline{Q}$.

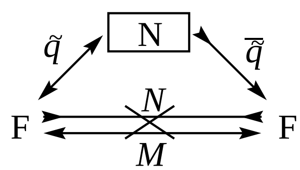
Finally, the size of the dual gauge group is obtained by matching the anomalies on the global groups. Before dualizing there were N fundamentals for each of the global groups, now there are F fundamentals corresponding to the meson M. Thus we need $F - N$ antifundamentals from the dual quarks. The anomaly matching condition can be summarized as a simple rule: the number of arrows entering any group minus the number of arrows leaving it has to be preserved under the duality transformation. The notation is a simple book keeping device for the non-abelian symmetries' anomalies but the $U(1)$ symmetries are less transparent, one might want to keep track of them by indicating the charges on the lines in the diagram. However, the orginal analysis of Seiberg [1] guarantees that the $U(1)$ anomalies match between the electric and dual theories.

It is also possible to identify a rule that determines the superpotential of the dual theory from the diagram. Whenever a closed cycle of lines, with the arrows on the lines oriented in such a way that all the indices can be contracted (i.e. one arrow has to be going into a group while the other one going out), is generated in a diagram, then the symmetries allow a superpotential term to be written. The contraction of indices that corresponds to the flow of arrows is that using δ^i_j tensors. In the absence of the superpotential term, the global symmetries of the dual theory would be much larger. In particular, the global symmetries of the meson field would not have to be

the same as the dual quarks'. The superpotential term is required to break unwanted global symmetries whenever there is a closed cycle of lines.

A new situation arises when we "dualize" our last dual again. We get an $SU(N)$ theory with new quarks and a meson $N = q\bar{q}$ and a term in the superpotential coupling the new quarks to the meson. The superpotential term inherited from the first dual maps onto a mass term for M and N. The complete superpotential is then $W = N(M - \tilde{\tilde{q}}\tilde{\tilde{q}})$. After integrating out the massive fields with their equations of motion we recover the original theory of SQCD with vanishing superpotential. For details see [1].

In our diagramatic notation all this is very simple. We "dualize" the $SU(F-N)$ gauge group by inverting the outside arrows on the quark lines and draw the new meson field N with the arrows given by the direction of the arrows of the quarks q and $\bar{q}$.



(2.5)

Note that the lines corresponding to the mesons M and N are parallel with opposite arrows. This indicates that the symmetries allow (and require) a mass term for these two fields. Thus, these fields are not present in the infrared and should be integrated out. We denote the mass term by crossing out the two lines. Note that after removing the massive fields we recover the original diagram (2.2) with no superpotential because there is no closed cycle in the diagram. In the following section we study duals with more complicated superpotentials involving non-renormalizable terms as well. We will see that the notation keeps track of these as well.

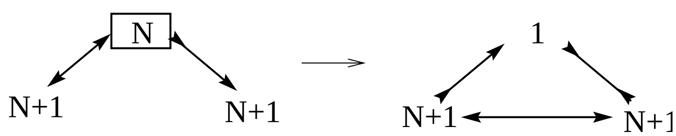
Let us summarize the "Duality Rules" in a few steps:

- the global symmetries of the model are unchanged, whilst the gauge group is replaced by a dual gauge group
- "dualize" the quark lines by reversing the arrows on the global groups
- draw a meson line corresponding to every pair of in- and out-going lines through the gauge group. The arrows on the meson line indicating its representations under the global groups are identical to the arrows on the orginal quark lines
- determine the size of the dual gauge group by matching the anomalies of the global groups
- each closed cycle of lines with matching arrows corresponds to a superpotential term

- parallel lines with opposite arrows correspond to massive fields and should be "integrated out" by simply dropping them

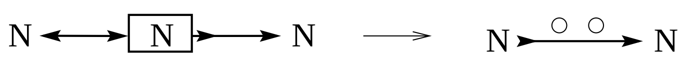
2.1 Special Cases: $F \leq N+1$

As discussed above, when F falls below $N+2$ the theory does not have a dual. Seiberg has again provided the correct low energy description of these theories[6]. Duality Diagrams may also be used for these cases though the notation becomes more opaque with decreasing F. For $F = N+1$, the correct low energy theory contains the usual mesons and fundamental baryon and anti-baryon superfields that transform under the global symmetry groups as the dual quarks did for larger F. The Duality Rules apply as before except that the dual gauge group is replaced by a global $U(1)$. The superpotential term $bM\overline{b}$ is generated, and hence the meson and baryon fields form a triangle on the Duality Diagram. In addition, there is also the "instanton term" $det(M)$ which is not encoded in the diagram in any apparent manner, and has to be remembered separately.



(2.6)

For $F = N$, there are again mesons transforming under the global groups but the baryon fields are now singlets under all non-abelian groups. There is a quantum constraint $\det M - B\overline{B} = \Lambda^{2F}$ on the moduli space. We indicate the baryons and their constraint by drawing them as two dots over the corresponding meson line

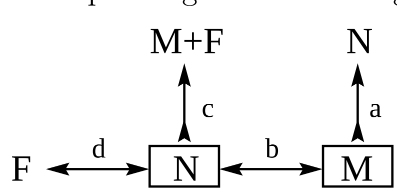


(2.7)

For $F < N$, the theory has no stable vacuum[7], thus there is no dual diagram to draw. For $F = 0$, the pure glue theory confines with no massless degrees of freedom. In the diagram one just erases the gauge group.

2.2 Example: $SU(N) \times SU(M)$

Armed with this set of rules let us now work through a non-trivial example first studied in [3]. It is a theory with two SU gauge groups, various fundamentals and no superpotential. We first define the theory with the use of our diagrams and then, for comparison, also give the corresponding table of Young tableaux.



(2.8)

In the table the first two groups are the gauge symmetries, the other three are global

Field	$SU(N)$	$SU(M)$	$SU(F)$	$SU(M+F)$	$SU(N)$
a	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
b	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
c	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
d	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$

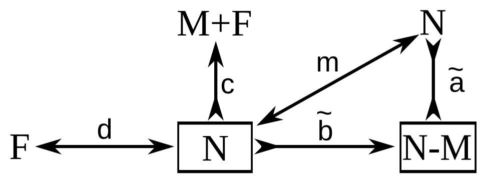
(2.9)

We will construct duals to this theory by dualizing the two gauge groups one at a time while treating the other as a global "spectator" of the duality transformation. The justification for doing so relies on holomorphy of the infrared superpotential, W, as a function of parameters of the theory. The supersymmetric vacua are determined by solving $V = |dW/d\phi|^2 = 0$. If such a vacuum exists in a patch of parameter space, then a corresponding vacuum exists for all values of parameters (except possibly some discrete points, such as $\Lambda = 0$). This follows because for a vacuum to be lifted at some critical parameter value would require non-holomorphic behavior as V would have to jump from zero. Therefore, there are no phase transitions, and the moduli space of vacua is independent of the parameters of the theory. This allows one to take the scale of one of the gauge groups to be very large compared to all the other scales. Then all other gauge interactions can be treated as small perturbations. Thus one can give a dual description of this theory by just dualizing the gauge group with the large scale, treating the others as spectators. It then follows from our holomorphy argument that the duality must also hold when the scale of the dualized gauge group is not large compared to other scales in the problem. The global anomalies of the duals obtained in this way are guaranteed to match because the global groups are subgroups of the global symmetry group of SQCD. For explicit consistency checks on dualities of product group theories see [3].

The conjecture which we want to verify is that when one alternatingly dualizes the gauge groups of our example five times, one obtains the original group back [3]. We impose the constraint $M - 1 < N < F + M + 1$ because the duals which we will encounter along the way only exist for this range of F, M, and N. These duality transformations can be performed using tables of Young tableaux, but the tables involved are large and the process is cumbersome. Instead, we are going to perform all the transformations using diagrams and the Duality Rules. The effort is considerably less. For comparison we give the tables and superpotentials for each of the duals in an appendix.

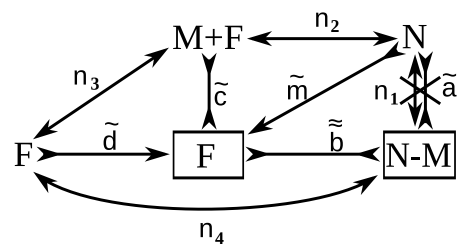
First, we dualize the $SU(M)$ gauge group. Following our rules, we add a meson field corresponding to the gauge invariant made from a quark and an antiquark of the $SU(M)$. Then we dualize the two quark lines by reversing the outside arrows. The

dual gauge group is $SU(N-M)$ with the following diagram



(2.10)

Note that there is a closed cycle with a corresponding superpotential term $W = m\tilde{a}\tilde{b}$. Dualizing the $SU(N)$ we arrive at



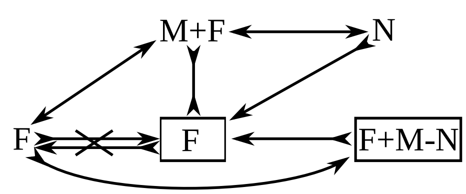
(2.11)

In this step four meson fields had to be added corresponding to the four possible ways of pairing up the two ingoing and two outgoing arrows of the $SU(N)$ gauge group. This created four new closed cycles with four corresponding superpotential terms.

$$W = n_1\tilde{a} + n_1\tilde{\tilde{b}}\widetilde{m} + n_2\tilde{c}\widetilde{m} + n_3\tilde{c}\tilde{d} + n_4\tilde{d}\tilde{\tilde{b}}. \quad (2.12)$$

Note that the Yukawa coupling that was present from the previous step has been turned into a mass term because two of the fields were replaced by a composite meson. Integrating out the massive fields yields the theory with n_1 and $\tilde{a}$ dropped and replaced by their equations of motion in the superpotential. In the following steps, we will not explicitly write down superpotentials since the diagramatic notation automatically keeps track of them. If in doubt, the reader can confirm this from the appendix where we give the complete superpotentials.

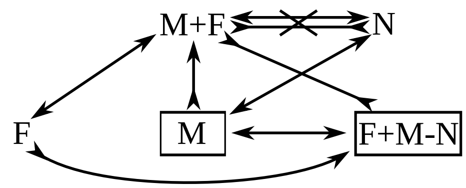
We now dualize the $SU(N-M)$ and obtain



(2.13)

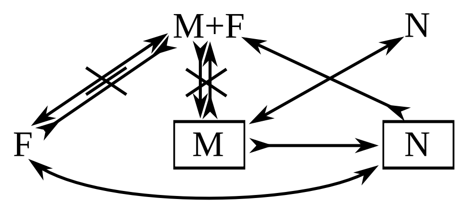
Note that integrating out the massive fields in this step generates a four-cycle in the diagram which corresponds to a quartic term in the superpotential. Dualizing $SU(F)$

we find



(2.14)

And finally dualizing $SU(F+M-N)$



(2.15)

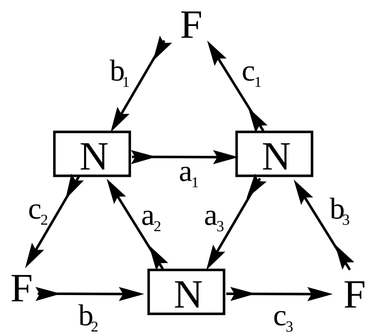
we return to the original theory except that the $SU(M)$ gauge group has been replaced by its complex conjugate, and the two gauge groups changed places. Integrating out all the massive fields in the last duality step has set the superpotential to zero, as required.

3 Loop Models

To demonstrate the power of the Duality Diagrams we next turn to the more complicated examples of diagrams with "loops" of matter fields and gauge groups which have not been studied before. The models we consider have k $SU(N)$ gauge groups, k $SU(M)$ global groups and $3k$ chiral superfields. These models turn out to be of interest in their own right since they do not possess a conformal phase. This is in contrast to the naïve expectation that as long as one chooses flavors and colors such that all gauge groups taken individually would be at a perturbative fixed point, then the whole theory should be at a perturbative (conformal) fixed point. In our model this kind of reasoning fails dramatically.

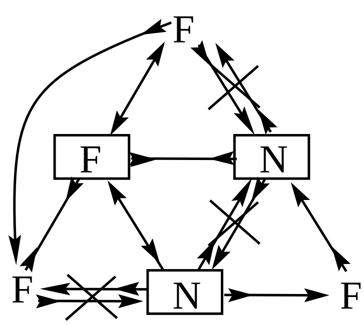
3.1 The Triangle, $k = 3$

The simplest non-trivial example has $k = 3$. The model is then



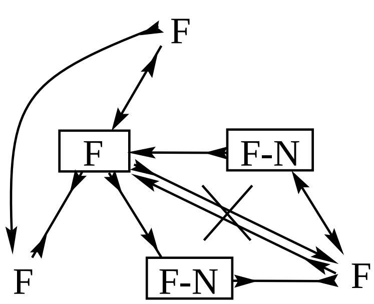
(3.1)

We include all four Yukawa couplings compatible with the symmetries (one associated with each triangle in the diagram). Dualizing one of the gauge groups leads to



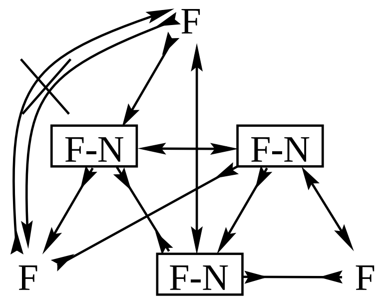
(3.2)

Next, dualizing the two remaining $SU(N)$ gauge groups in one step, we find



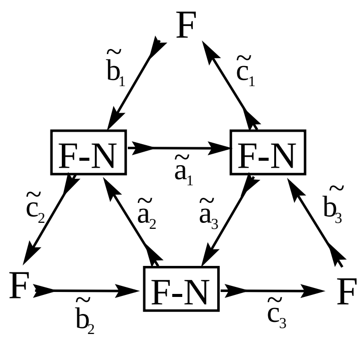
(3.3)

Finally, dualizing the $SU(F)$ gauge group gives



(3.4)

This dual has the same form as the orginal theory. To make this explicit we first interchange the two gauge groups on the right of the diagram and then complex conjugate them.



(3.5)

The dual and the electric theory are identical except that the dual gauge group is $SU(F-N)^3$ instead of $SU(N)^3$.

Our orginal model is in a weakly coupled "free electric" phase when it's number of flavors $(F+N)$ is greater or equal than three times the number of colors N, that is when $F \geq 2N$. Similarly, the dual magnetic theory is in it's "free magnetic" phase when $F \leq 2N$. At $F = 2N$ it appears that we have two different infrared-free descriptions of the same theory. Clearly, this would be a contradiction. However, everything is consistent, because for $F = 2N$ the theory and its dual are identical.

Thus we have an infrared free description for any $F \geq N+2$. This result is somewhat surprising since other exactly solved supersymmetric theories have a conformal "window" for a certain range of flavors. For example, SQCD has a conformal phase for $3/2N < F < 3N$ in which neither the electric theory nor its dual are free in the infrared.

If we did not have our dual description we might have been tempted to argue that there should be such an interacting perturbative fixed point in the large N and F limit for F just below $2N$. Then each of the gauge groups taken individually would be at a perturbative Banks-Zaks fixed point[8]. However, it does not follow that the theory as a whole is perturbative in this description as well. The problem is that all

of the chiral superfields obtain negative anomalous dimensions through the action of the gauge groups. Assuming that the theory is at a non-trivial fixed point implies that the anomalous dimension of a composite chiral operator is equal to the sum of the anomalous dimensions of its constituents (this follows from the superconformal R-symmetry [1]), and thus the dimensions of the Yukawa couplings are less than three, rendering the Yukawa couplings relevant. Therefore the Yukawa couplings are large in the infrared and a perturbative analysis like the one we attempted is invalid.

A single group in the model discussed here can be placed in an approximate conformal phase by taking its scale Λ much greater than those of the other two groups. However, eventually the other two groups become strong at low energies and contribute to anomalous dimensions of the fields which appear in the beta function of the original group. This can be seen in the duals of the theory where dualizing one group leads to a change in the number of flavors of the other groups. The gauge and Yukawa couplings interfere so that the model does not have a non-trivial fixed point in the infrared.

Another indication that the theory is not at a conformal fixed point, comes from the fact that for $F \neq 2N$ the superpotential couplings break all non-anomalous R-symmetries. Thus there is no candidate for the anomaly free superconformal R-symmetry which is part of the superconformal algebra[1].

Even though the construction of the dual via a series of elementary dualities guarantees consistency at each step, it is interesting to perform some consistency checks on the final dual. In addition to the non-abelian $SU(F)$ flavor symmetries, the theory also possesses three non-anomalous $U(1)$ symmetries. As a first consistency check, we find that all global anomalies match. Furthermore, we can define a map identifying each non-vanishing gauge-invariant operator of the original theory with a partner in the dual. For the meson fields this operator map is trivial, mapping the mesons b_1c_2, b_2c_3, b_3c_1 onto $\widetilde{b_1}\widetilde{c_2}, \widetilde{b_2}\widetilde{c_3}, \widetilde{b_3}\widetilde{c_1}$, respectively. An interesting consistency check arises when we integrate out flavors in one of the theories. Let us, for example, add mass terms for one set of flavors in the electric theory. The superpotential is then

$$W = a_1a_2a_3 + \sum_{i=1}^{3} a_ib_ic_i + b_1c_2 + b_2c_3 + b_3c_1, \tag{3.6}$$

where the flavor indices are summed over in the $a_ib_ic_i$ terms, whereas the mass terms are only for the F'th flavors. The equations of motion for the massive fields yield two different branches of vacua. The "mass branch" with vanishing vacuum expectation values for the massive b_i and c_i simply reduces the number of flavors by one and leaves the gauge groups unchanged. The other branch, the "Higgs branch", has expectation values $\langle a_1a_2a_3\rangle = 1$ and non-zero expectation values for the b_i and c_i. In this branch, the gauge groups are broken to $SU(N-1)$ and the number of flavors is reduced to

[1]This argument is not completely rigorous because the superconformal R-symmetry might be an accidental symmetry of the infrared.

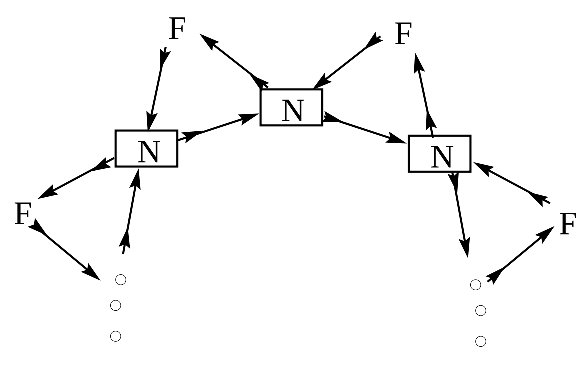
$F-1$. It is interesting to see how this is realized in the dual. The mass terms simply map onto corresponding mass terms $\widetilde{b_1}\widetilde{c_2}+\widetilde{b_2}\widetilde{c_3}+\widetilde{b_3}\widetilde{c_1}$. Again, there are two branches. But the correspondence between the branches in the two descriptions is non-trivial: the "mass branch" of the electric theory is mapped onto the "Higgs branch" of the magnetic theory and vice versa. This non-trivial mapping is required to preserve the duality $N \leftrightarrow \widetilde{N} = F - N$ and $F \leftrightarrow \widetilde{F} = F$ under the mass perturbation.

We conclude this section by briefly stating results for the theories with $F < N+2$ which are very similar to results in SQCD with the corresponding numbers flavors[6]. For $F = N + 1$, the dual has no gauge group and the theory is confining. In the infrared the theory is described by mesons and baryons interacting via a confining superpotential. The diagrams then don't enable one to keep track of the determinant terms in the superpotential. This is analogous to SQCD with $F = N + 1$. For $F = N$, the theory confines with chiral symmetry breaking, again the discussion is very similar to SQCD with $F = N$. Finally, for $F < N$, nonperturbatively generated superpotentials lift the potential at the origin. The vacuum is stabilized by our Yukawa couplings, and we find supersymmetric vacua at non-zero expectation values for the fields. The remaining unbroken gauge groups $SU(N - F)$ lead to gaugino condensation.

3.2 Loops with $k \geq 3$

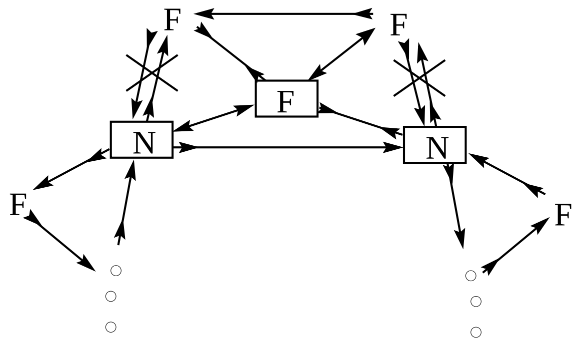
The model may be extended to include k $SU(N)$ gauge groups in the loop and k $SU(F)$ global groups in the same pattern as the model above. We again include Yukawa interactions for all the superfields transforming under the global symmetry groups. The Yukawa interaction for the central loop of the model in (3.1) is now replaced by a (non-renormalizable) dimension k operator involving each of the fields in the loop.

We now present a simple inductive proof to show that these models also have duals of the same loop form but with dual gauge group $SU(N - F)$. The case $k = 3$ provides the first step for our induction. Assume that the case $k-1$ has been proven. Now, consider a segment of the loop of the k'th model



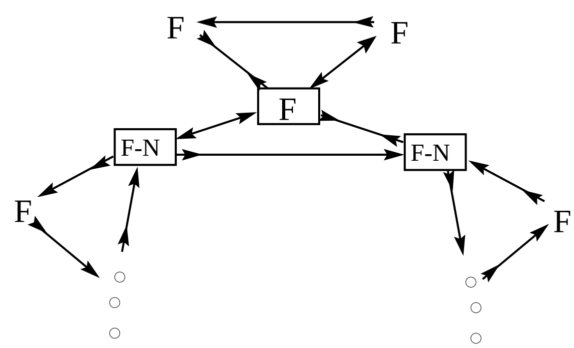
(3.7)

We begin by dualizing an $SU(N)$ gauge group



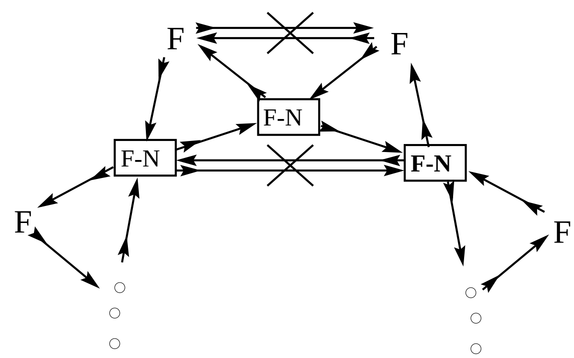
(3.8)

The resulting dual has the form of a loop model with $k-1$ $SU(N)$ gauge groups but where one $SU(F)$ global group is the dual gauge group just produced. In addition, there are some superfields that do not interact with the rest of the loop. Then, treating the $SU(F)$ as a "spectator", we can apply the dual for $k-1$ and obtain



(3.9)

Now dualizing the $SU(F)$ group gives



(3.10)

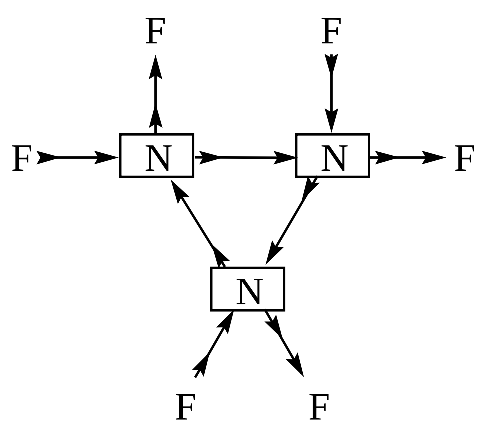
and we see that the k'th loop has a dual of the form advertised. In total, to reach this dual description it takes $2(k-1)$ separate steps of dualizing subgroups. The

larger k loop models also have the property that the electric description is infrared-free for $F \geq 2N$, whereas the magnetic description is free for $F \leq 2N$. They have no conformal phase.

The case $k = 2$ is special, since the "Yukawa coupling" for the central loop is now a mass term. On integrating out the massive multiplets the Yukawa terms of the fields transforming under the global groups generate a quartic interaction involving the remaining fields. The two gauge groups may then be dualized, and the two resulting mesons obtain masses from the operator map of the quartic term. The dual with gauge groups $SU(F-N)$ is similar to the original theory again. However, the range of flavors for which the two descriptions are free differ from the $k > 2$ cases. The electric theory is free for $F \geq 3N$ whereas the magnetic theory is free for $F \leq 3/2N$. For $3/2N < F < 3N$ the theory flows to an interacting fixed point in the infrared.

3.3 Related Models

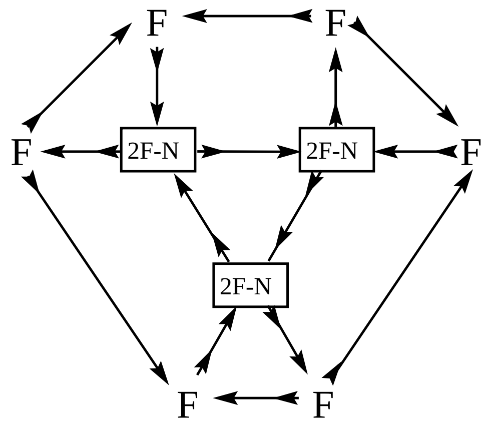
The models and their duals take a different form if we remove the Yukawa couplings between the superfields transforming under the global groups. Removing these interactions increases the global symmetry of the model which then takes the form (for $k = 3$)



(3.11)

The model is again brought to an interesting dual by dualizing the subgroups $2(k-1)$ times. This dual has k $SU((k-1)F - N)$ gauge groups, $2k$ $SU(F)$ global groups,

and additional mesons connecting the global groups. For $k = 3$ the dual is



(3.12)

Unlike its sister (3.1) this model does have a conformal phase for $2N/3 < F < 2N$. In the general case, the conformal range extends from $F = 2N$ down to $F = 2N/(2k-3)$. Again, this is different from what one would have expected from treating each of the gauge groups separately. Then one would have falsely concluded that the conformal range is $N/2 < F < 2N$.

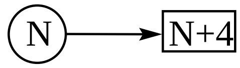
4 Duality Diagrams for SO and Sp groups and tensors

So far, we have presented our notation only for SU groups with matter transforming in the fundamental and antifundamental representations. The notation is easily extended to deal with SO and Sp gauge groups. We simply append an index indicating the different groups. For an $Sp(2N)$ gauge group[2] we write $\boxed{\mathbf{2N}}_{\mathbf{Sp}}$, and a global $SO(F)$ symmetry would be denoted as $\mathbf{F_{SO}}$. Since the representations of Sp and SO are (pseudo)real we do not need to put any arrows on the ends of lines connected to these groups. For example, a field transforming as a fundamental under an $SO(N)$ gauge group and a fundamental under a global $SU(F)$ would be $\boxed{\mathbf{N}}_{\mathbf{SO}} \longrightarrow \mathbf{F}$. Sometimes, to avoid confusion, it might be useful to use a subscript to denote SU groups as well.

When working with these groups one often encounters fields transforming as symmetric and antisymmetric two-index tensors. To denote them we also use lines but we put circles with their cubic anomaly coefficient on the lines. The cubic anomaly is sufficiently unique to distinguish almost all tensor representations.In those cases where it is not sufficient (for example Sp and SO gauge groups with only (pseudo)real representations where these indices all vanish) the Young tableaux may be used instead to denote the representation. This enables one to quickly calculate the cubic anomalies associated with any of the non-abelian groups in the diagram. For example,

[2]In our convention, $Sp(2N)$ is the group whose fundamental representation is $2N$ dimensional. In the literature, this group is sometimes referred to as $Sp(N)$.

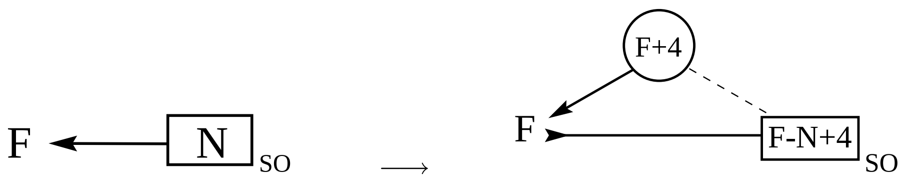
a field transforming as an antisymmetric two index tensor under a gauged $SU(N+4)$ would be



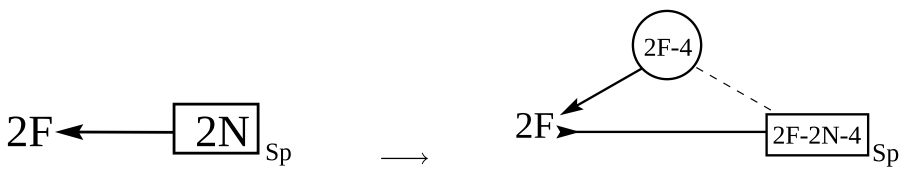
(4.1)

To retain the connection between superpotential terms and closed loops in more complicated models is difficult. One possibility is to introduce an extra bit of notation for higher dimension representations. Every time a meson which is a higher dimension representation of a global group is generated by a duality transformation a dotted line may be included linking it to the gauge group that confined its constituents. The dotted line now closes a cycle indicating a superpotential term involving the meson and the other fields in the loop, though the precise form of the superpotential term is not explicit from the diagram. It may be necessary to keep track of the superpotential separately.

In this notation, the duals for SO and Sp gauge theories with fundamentals [9], respectively, are given by



(4.2)



(4.3)

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A Tables for the SU(N) × SU(M) Model

In this appendix, we give tables of representations and superpotentials for the model given in section 2.2. We denote fields that have been "dualized" once by a tilde, and fields that have been dualized i times by an upper index $^{(i)}$. After each duality step we list the full particle content and superpotentials including all fields, followed by the superpotential obtained by integrating out the massive fields.

Field	$SU(N)$	$SU(M)$	$SU(F)$	$SU(M+F)$	$SU(N)$
a	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
b	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
c	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
d	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$

(A.1)

$$W = 0 \tag{A.2}$$

Field	$SU(N)$	$SU(N-M)$	$SU(F)$	$SU(M+F)$	$SU(N)$
$\tilde{a}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
$\tilde{b}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
c	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
d	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$
m	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$

(A.3)

$$W = m\tilde{a}\tilde{b} \tag{A.4}$$

Field	$SU(F)$	$SU(N-M)$	$SU(F)$	$SU(M+F)$	$SU(N)$
$\tilde{a}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
$b^{(2)}$	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\tilde{c}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\tilde{d}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$
$\tilde{m}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
n_1	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\square$
n_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
n_3	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
n_4	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$

(A.5)

$$W = n_1\tilde{a} + n_1 b^{(2)}\tilde{m} + n_2\tilde{c}\tilde{m} + n_3\tilde{c}\tilde{d} + n_4\tilde{d}b^{(2)} \quad \longrightarrow \quad n_2\tilde{c}\tilde{m} + n_3\tilde{c}\tilde{d} + n_4\tilde{d}b^{(2)} \tag{A.6}$$

Field	$SU(F)$	$SU(F+M-N)$	$SU(F)$	$SU(M+F)$	$SU(N)$
$b^{(3)}$	$\square$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\tilde{c}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\tilde{d}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$
$\tilde{m}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
n_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
n_3	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
$\tilde{n}_4$	$\mathbf{1}$	$\square$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$
p	$\overline{\square}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$

(A.7)

$$W = p\tilde{d} + n_2\tilde{c}\tilde{m} + n_3\tilde{c}\tilde{d} + pb^{(3)}\tilde{n}_4 \quad \longrightarrow \quad n_2\tilde{c}\tilde{m} + n_3\tilde{c}b^{(3)}\tilde{n}_4 \tag{A.8}$$

Field	$SU(M)$	$SU(F+M-N)$	$SU(F)$	$SU(M+F)$	$SU(N)$
$b^{(4)}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$c^{(2)}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
$m^{(2)}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
n_2	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
n_3	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
$\tilde{n}_4$	$\mathbf{1}$	$\square$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$
q_1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$
q_2	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$

(A.9)

$$W = q_1n_2 + q_2n_3\tilde{n}_4 + q_1c^{(2)}m^{(2)} + q_2b^{(4)}c^{(2)} \quad \longrightarrow \quad q_2n_3\tilde{n}_4 + q_2b^{(4)}c^{(2)} \tag{A.10}$$

Field	$SU(M)$	$SU(N)$	$SU(F)$	$SU(M+F)$	$SU(N)$
$b^{(5)}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$c^{(2)}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
$m^{(2)}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
n_3	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
$n_4^{(2)}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\tilde{q}_2$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
r_1	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$
r_2	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$

(A.11)

$$W = r_1n_3 + r_2c^{(2)} + r_1n_4^{(2)}\tilde{q}_2 + r_2b^{(5)}\tilde{q}_2 \quad \longrightarrow \quad 0 \tag{A.12}$$

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